

Dear AMHS Accelerated Pre-Calculus Student:

GSE Accelerated PreCalculus is a fast-paced, college-preparatory mathematics course that will also prepare you to take Calculus in the year following successful completion. You should be aware that the course requires substantial work and a great work ethic. You will be challenged to use your writing, reading comprehension, and critical thinking skills as well as your mathematical skills in order to solve problems. Your success in this course will be the result of your consistent attention to reading, discussion, and practice.

During the semester, we will be exploring many new concepts. However, because of the pace and rigor of the course, there will not be a lot of time to spend reviewing topics that you were exposed to in your previous math courses. The following problems serve as a review of topics taught in those courses. Questions have been organized according to the GSE Accelerated Pre-Calculus units for which you will need to apply the respective concepts.

Directions:

- Work all of the problems neatly and legibly on notebook paper, unless indicated otherwise.
- Be sure to number your work and CIRCLE your answers.
- **You must show your work for ALL problems, including formulas used. If there is no work that can be shown, you should provide a mathematically reasonable explanation.**
- Be sure to write **all final answers** on the **answer sheet**.

The following internet resources may be of some assistance to you as you work through these questions: <http://themathpage.com/>, <http://www.purplemath.com/>, <http://www.sosmath.com/>, <https://www.khanacademy.org/>, .

The completed packet represents **your first assignment in Accelerated Pre-Calculus**. It will be graded for completion and accuracy. All work and answer sheets are due on the first day of class which is **August 18, 2017** .

Although we will begin the course with new material, during the first week of the semester there will be time for you to ask any questions about difficulties you encountered while working the problems. Therefore, be sure to make a note of the problems that you have difficulty solving.

We look forward to working with each of you in the upcoming school year. Please feel free to contact the department chair, Dierdre_M_Watkins@dekalbschoolsga.org if you have any questions. Also, understand that Ms. Watkins is on Summer vacation and may not answer immediately.

Give it your best!

Ms. Dierdre Watkins & Mr. Martin Gilbert
Accelerated PreCalculus Teachers

**Accelerated CCGPS Pre-Calculus
Summer Work Packet**

Unit 4: Conics

MGSE9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

A. Parent Graphs – Sketch the graphs of the following functions without using a graphing calculator.

You should be familiar enough with these “parent” graphs to be able sketch them from memory. If you are unsure about how to graph the functions, make a “t”-chart of x - and y - values. Label important features of each graph, such as intercepts, minimums, and maximums. Label the scales and axes of the graphs.

1. $f(x) = x$

2. $f(x) = x^2$

3. $f(x) = x^3$

4. $f(x) = \sqrt{x}$

5. $f(x) = |x|$

6. $f(x) = \frac{1}{x}$

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

B. Transformations of Basic Functions – Describe the transformations from the parent function, $f(x)$ to the function $g(x)$ such as vertical shifts, stretches, and shrinks, as well as reflections across the x - and y - axes) for each of the following functions.

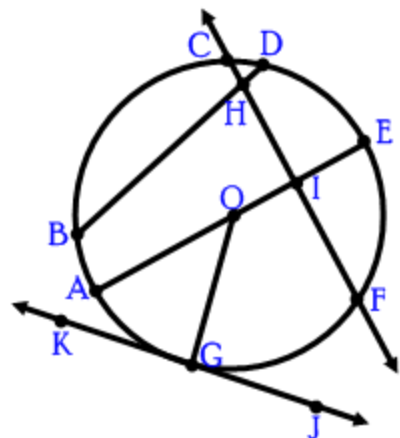
7. $f(x) = x^3$; $g(x) = -2(x - 3)^3 + 1$

8. $f(x) = \sqrt{x}$; $g(x) = \frac{1}{3}\sqrt{x+1} - 2$

MGSE9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

C. Circle Relationships – Given the diagram on the right, indicate the point, segment, or line (by name) for each of the following:

9. Center
10. Diameter
11. Two chords that are not diameters
12. Secant line
13. Tangent line
14. Point of tangency
15. All radii
16. What is the measure of $\angle JGO$? How do you know?



MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

D. Equations of Circles

Given the information below, write the equation of the circle.

17. Center at (0, 0); Radius of 4

18. Center at (2, -5); Radius of 3

19. Center at (-3, -6) and passes through (-4, 8)

Write the equation in standard form by completing the square. Identify the center and radius of the circle.

20. $x^2 + y^2 + 16x - 22y - 20 = 0$

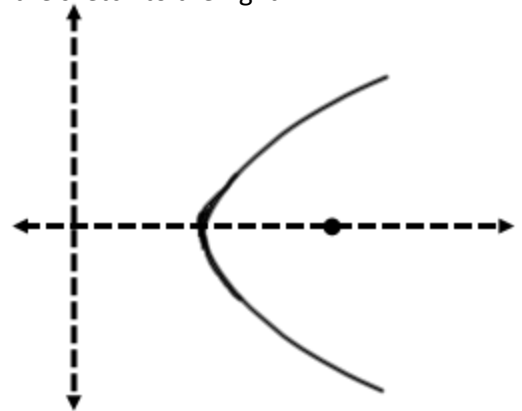
21. $x^2 + y^2 - 6x - 8y - 75 = 0$

MGSE9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

E. Equations of Parabolas – Label the following features, based on the sketch to the right.

22. Label these features on your answer sheet:

- Vertex
- Focus
- Axis of Symmetry
- Directrix
- The directed distance, **p** (label 2 different places)



Write the equation in standard form, given the following information.

23. focus (-6, 2), vertex (-6, -1)

24. focus (5, -2), vertex (9, -2)

25. vertex (-2, 4), directrix $y = 1$

26. $y = \frac{1}{4}x^2 + 3x + 6$ (Complete the square. Identify the vertex, focus, axis of symmetry, and directrix.)

Unit 1: Trigonometric Functions

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include even and odd functions from their graphs and algebraic expressions for them.

F. Identify Even and Odd Functions – Determine algebraically whether each function is *even*, *odd*, or *neither*. If even or odd, describe the symmetry of the graph of the function.

27. $f(x) = \frac{2}{x^2}$

28. $g(x) = 4\sqrt{x}$

29. $h(x) = x^5 - 2x^3 + x$

MGSE9-12.F.BF.4b (+) Verify by composition that one function is the inverse of another.

MGSE9-12.F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

MGSE9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

G. Inverses of Functions – See directions below.

30. Show that f and g are inverse functions: $f(x) = 18 - 3x$; $g(x) = 6 - \frac{x}{3}$.

(Show work on the answer sheet.)

31. Given the table of values for a relation below, provide a table for the inverse relation:

x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5

32. Find the inverse function of $f(x) = \sqrt{x-4}$. State any restrictions on its domain.

MGSE9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.

MGSE9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

H. Unit Circle – Visit <http://www.math.utah.edu/~gupta/MATH1060Fall2012/blankunitcircle.pdf>. Print and complete the blank unit circle and attach it to your answer sheet. You should include measurements in degrees and radians, as well as the ordered pair for each location. Use it to provide exact values (no decimals) for the following:

33. $\sin \frac{5\pi}{3}$

34. $\sec \frac{3\pi}{4}$

35. $\tan 180^\circ$

36. $\cos \frac{5\pi}{6}$

37. $\cot 0^\circ$

38. $\csc 135^\circ$

39. Explain how to change an angle that is measured in radians to degrees.

MGSE9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

I. Graphing Trigonometric Functions – Graph the following trigonometric functions for at least one period. Identify the important features of each graph: amplitude, phase shift, vertical shift, period, and midline (center line) for each graph. Also report any asymptotes.

40. $y = 2\cos(x - \pi)$

41. $y = -2\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right) - 1$

42. $y = \csc(2x)$

43. $y = 3\tan(x)$

Unit 2: Trigonometry of General Triangles

MGSE9-12.F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

J. Solving Trigonometric Equations – Solve the following equations for $0^\circ \leq \theta \leq 360^\circ$ and $0 \leq \theta \leq 2\pi$.

44. $\cos \theta + 1 = 0$

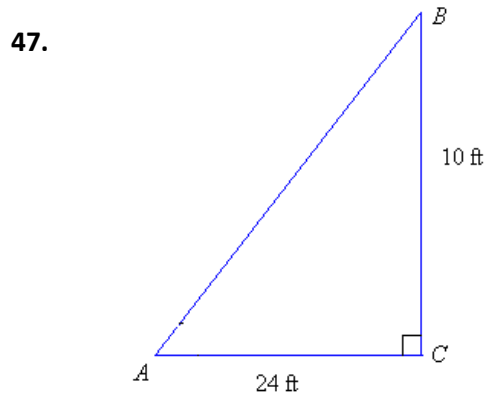
45. $2\sin \theta + \sqrt{3} = 0$

46. $\tan^2 \theta - 5 = 0$

Unit 3: Trigonometric Identities

MGSE9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

K. **Solving Right Triangles** - Solve the triangle shown below, giving the missing side lengths to the nearest tenth and all angles to the nearest degree.



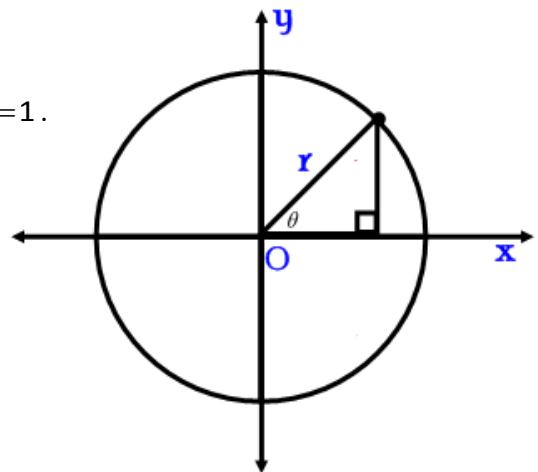
48. If $\tan \theta = \frac{1}{2}$, find the exact values of the five remaining trigonometric functions for θ , which is acute.

MGSE9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find $\sin A$, $\cos A$, or $\tan A$, given $\sin A$, $\cos A$, or $\tan A$, and the quadrant of the angle.

L. **Proving Trigonometric Identities**

49. Given the diagram on the right, prove that $\sin^2 \theta + \cos^2 \theta = 1$.
Hint: Use the Pythagorean Theorem.

(Show work on the answer sheet.)



50. Given $\cos \theta = -\frac{2}{3}$ and θ has its terminal side in Quadrant III, find $\sin \theta$.

Unit 5: Matrices

MGSE1.OA.3 Apply properties of operations as strategies to add and subtract.

MGSE3.OA.5 Apply properties of operations as strategies to multiply and divide.

M. *Properties of Numbers* – For each property listed below, provide an example which demonstrates the meaning of the property:

- | | |
|---|---|
| 51. Commutative property of addition | 56. Identity property of addition |
| 52. Commutative property of multiplication | 57. Identity property of multiplication |
| 53. Associative property of addition | 58. Inverse property of addition |
| 54. Associative property of multiplication | 59. Inverse property of multiplication |
| 55. Distributive property of multiplication | |

MGSE9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

N. *Solving Systems of Equations* – Solve the following system of equations (a) graphically, (b) by substitution, and (c) by elimination. Clearly indicate the solution in each case.

60. (a-c)
$$\begin{aligned} 3x - 2y &= -6 \\ x + y &= -2 \end{aligned}$$

Unit 6: Vectors

MGSE9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

MGSE9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.

O. *Reference Angles* – Sketch each angle. Then find its reference angle.

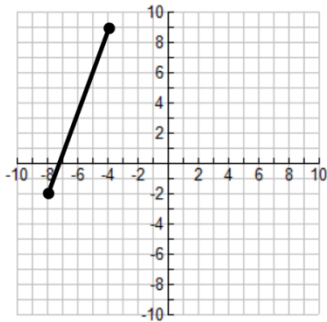
61. -150°

62. $\frac{3\pi}{4}$

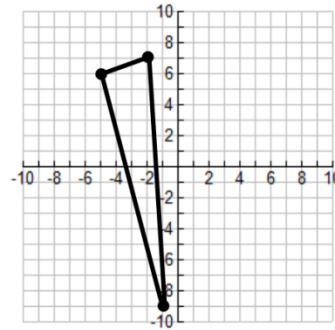
MGSE9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

P. Geometric Transformations - Draw the image of each figure, using the given transformation

63. Translation $(x, y) \rightarrow (x + 9, y - 8)$
Rotation 90° CCW about the origin



64. Translation $(x, y) \rightarrow (x + 4, y - 2)$
Rotation 180° about the origin.
Reflection about the line $y = x$



MGSE7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Q. Triangle Inequality Theorem - Two sides of a triangle have the following measures. Find the range of possible measures for the third side.

65. 5, 8

66. 14, 11

MGSE9-12.N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

MGSE9-12.N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

R. Operations with Complex Numbers – Simplify each expression.

67. i^{53}

68. i^{-12}

69. $(8 - 5i) - (4 - 2i)$

70. $(3 - 2i)^2$

71. $\frac{12 + 3i}{1 + 2i}$

MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

S. Rational Exponents – Simplify using properties of rational exponents. Leave answers in simplified radical form.

72. $\frac{b^{\frac{5}{4}} \cdot b^{\frac{3}{4}}}{b^{\frac{1}{4}}}$

73. $\frac{\sqrt[10]{32}}{\sqrt[6]{2}}$

74. $4096^{\frac{1}{4}}$

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

T. Distance Formula – Find the distance between the following points.

75. (4, 2) and (10, -4)

76. (-3, -5) and (-6, 0)

Unit 7: Probability

MGSE9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. ★

MGSE9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★

MGSE9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. ★

U. Probability of Events – Answer each of the questions below.

Use the following scenario to answer questions 77 and 78: In the state of Georgia, telephone numbers consist of 10 digits, including the area code. One of these area codes is 770. The first digit following the area code cannot be 1 or 0.

77. Find the number of possibilities for telephone numbers in area code 770.

78. How many different telephone numbers in this area code are possible if only even digits are used?

79. Suppose you roll two dice. Find the probability that the sum shown on the faces is greater than 8.

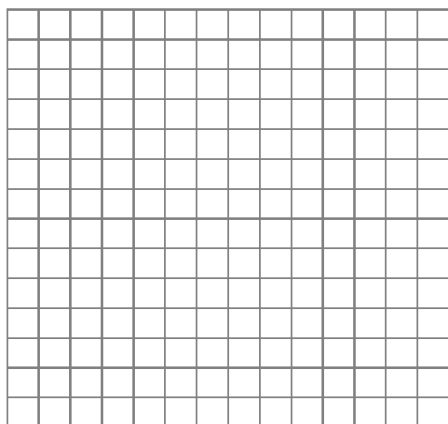
80. Find the probability of choosing a spade or a red card from a standard deck of 52 cards?

81. Find the probability of choosing a queen or a diamond from a standard deck of 52 cards?

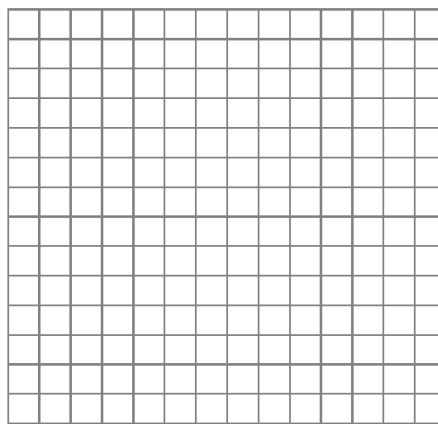
82. A bag contains 6 blue marbles and 10 red marbles. If you choose one marble at random, and then another marble at random, what is the probability that both marbles are blue?

A. Parent Graphs

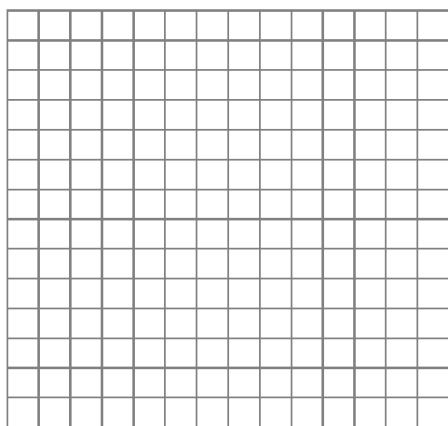
1.



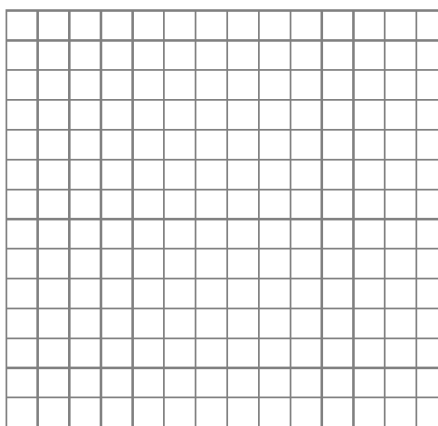
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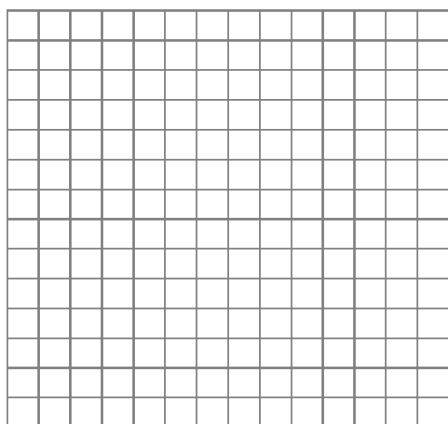
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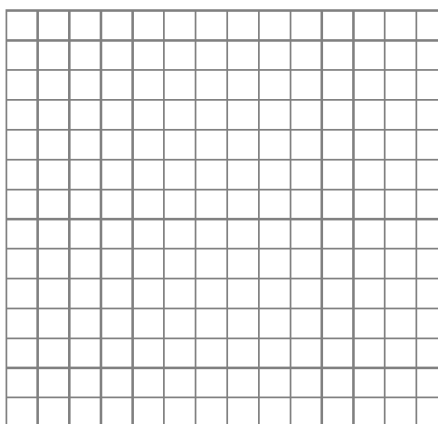
4.



5.



6.



B. Transformations of Basic Functions

7.

8.

C. Circle Relationships

9.

10.

11.

12.

13.

14.

15.

16.

D. Equations of Circles

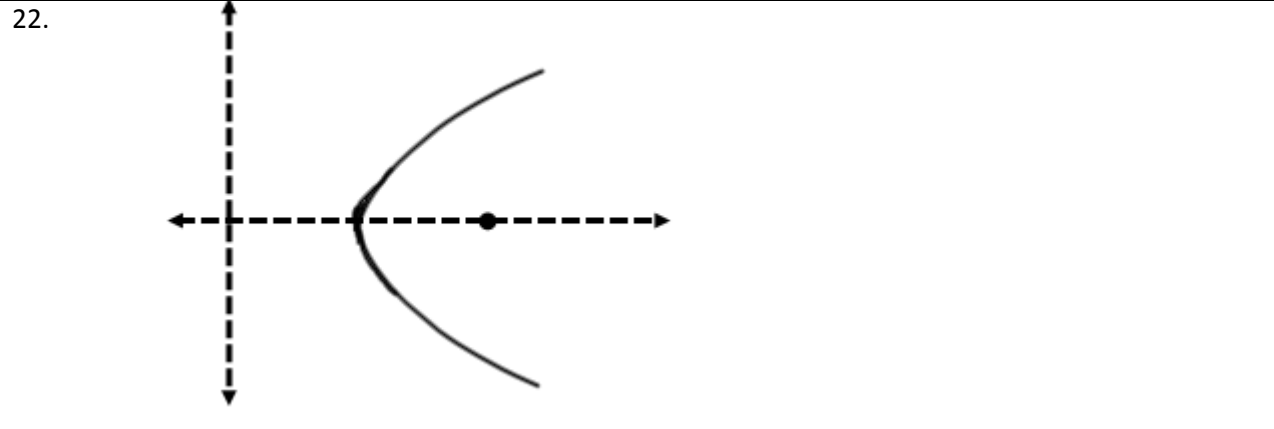
17.

18.

19.

20.

21.

E. Equations of Parabolas

23.

24.

25.

26.		
Equation:		Axis of symmetry:
Vertex:		Directrix:
Focus:		

F. Identify Even and Odd Functions

27.	28.	29.
-----	-----	-----

G. Inverses of Functions

30. **Show work here.**

31.

32.

$f^{-1}(x) =$

Domain Restrictions:

H. Unit Circle

33.	34.	35.
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36.	37.	38.
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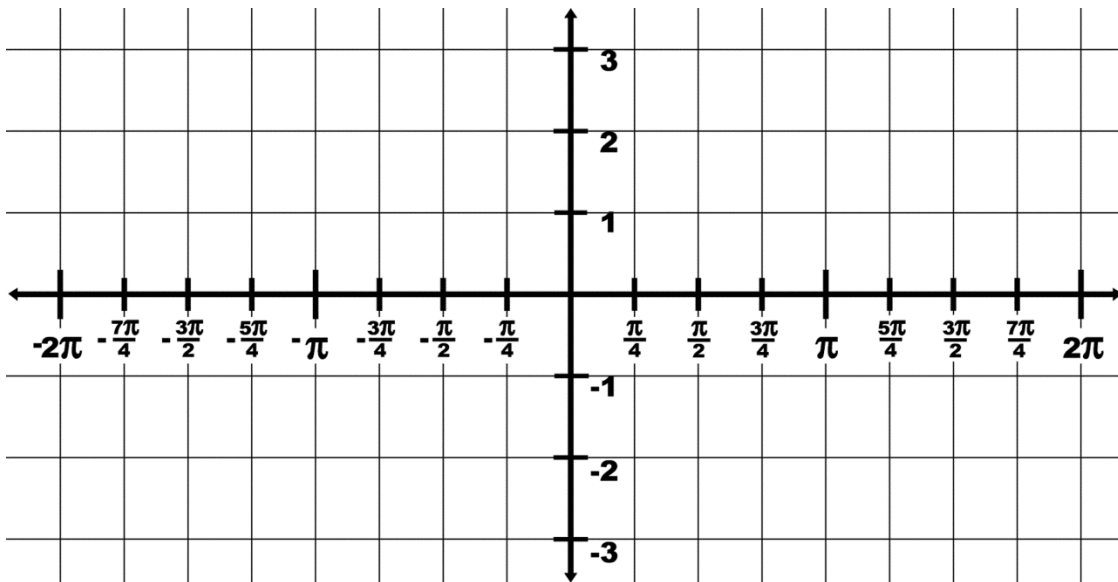
39.

I. Graphing Trigonometric Functions

40.

Amplitude:
Phase Shift:
Vertical Shift:

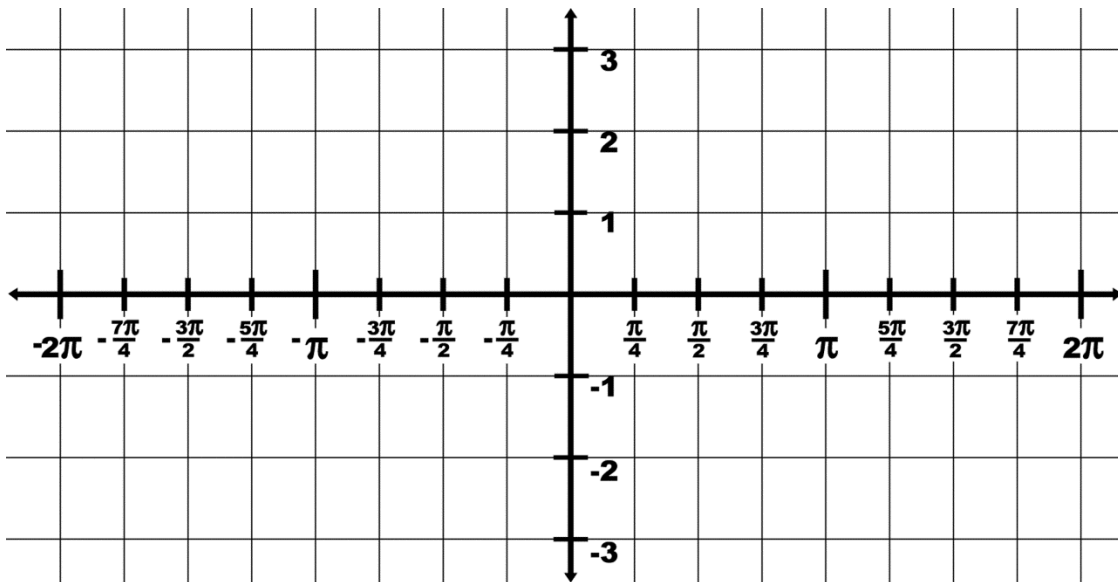
Period:
Midline:
Asymptotes:



41.

Amplitude:
Phase Shift:
Vertical Shift:

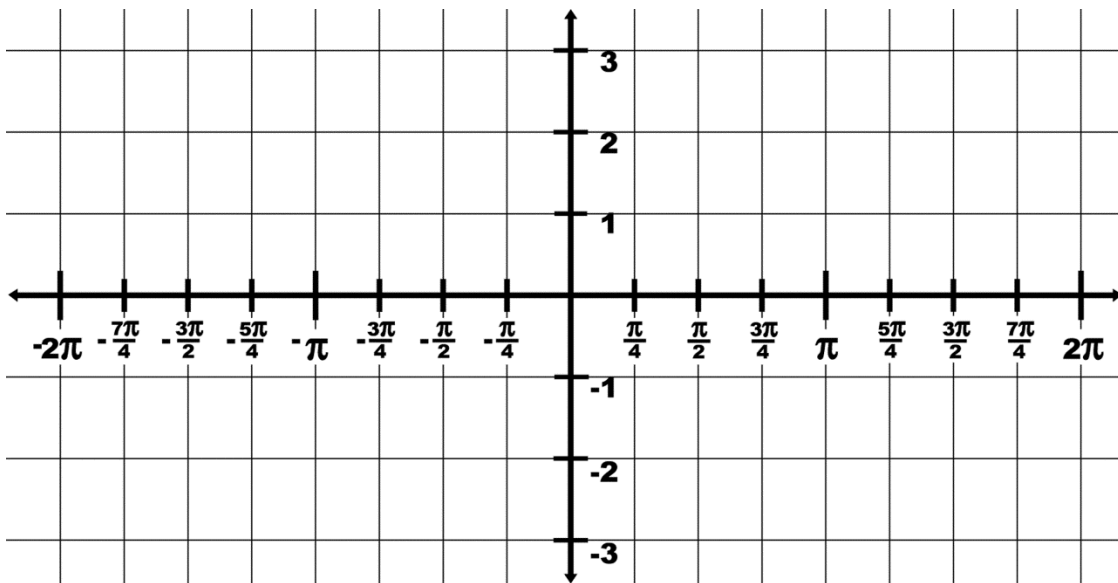
Period:
Midline:
Asymptotes:



42.

Amplitude:
Phase Shift:
Vertical Shift:

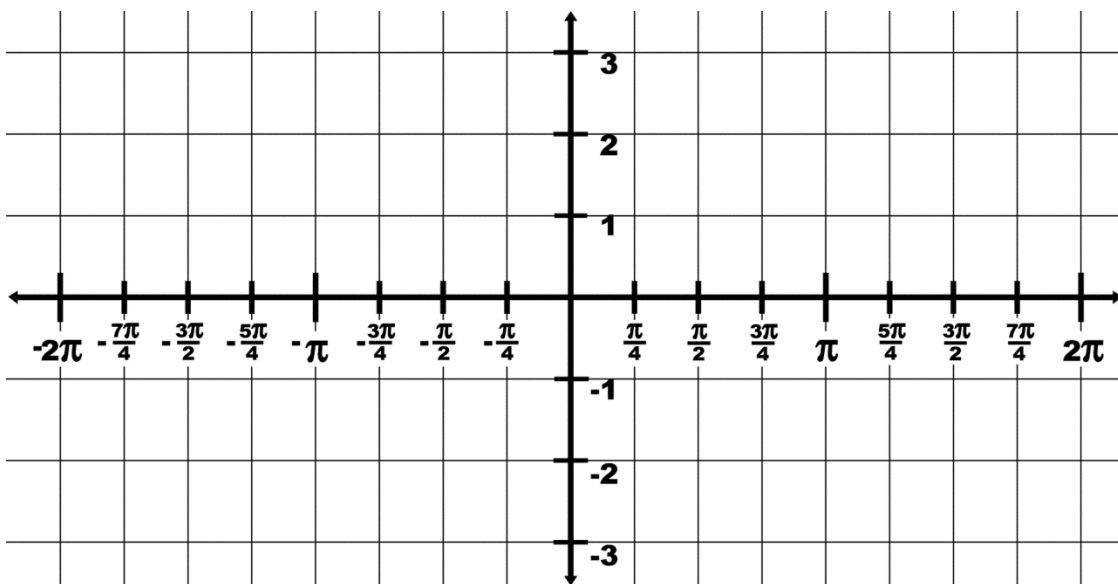
Period:
Midline:
Asymptotes:



43.

Amplitude:
Phase Shift:
Vertical Shift:

Period:
Midline:
Asymptotes:

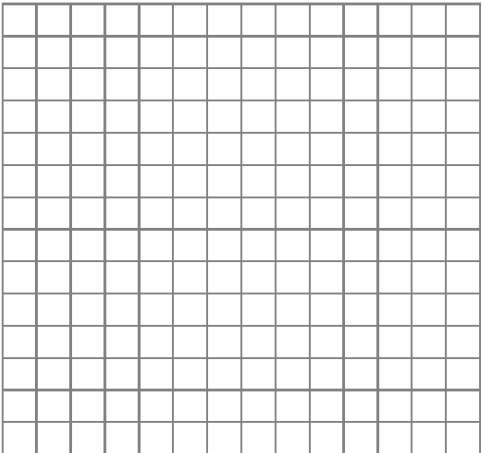


J. Solving Trigonometric Equations

44.

45.

46.

K. Solving Right Triangles		
47. AB = A = B =		48. $\sin \theta =$ $\cos \theta =$ $\csc \theta =$ $\sec \theta =$ $\cot \theta =$
L. Proving Trigonometric Identities		
49. Show work here.		50.
M. Properties of Numbers		
51.	52.	53.
54.	55.	56.
57.	58.	59.
N. Solving Systems of Equations		
60.a) 	60. b)	
	60. c)	

O. Reference Angles

61.

Drawing:

62.

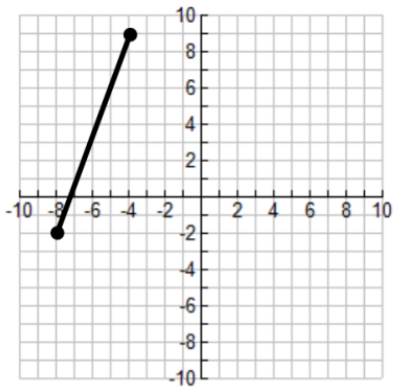
Drawing:

Reference Angle:

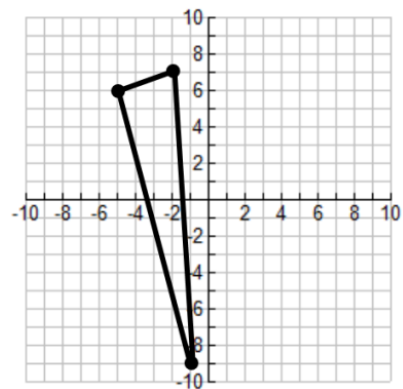
Reference Angle:

P. Geometric Transformations

63.



64.



Q. Triangle Inequality Theorem

65.

66.

R. Operations with Complex Numbers

67.

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69.

70.

71.

S. Rational Exponents

72.

73.

74.

T. Distance Formula

75.

76.

U. Probability of Events		
77.	78.	79.
80.	81.	82.