ARABIA MOUNTAIN HIGH SCHOOL

To: Regular Calculus and AP Calculus AB Students
From: Mr. Modenou and Mr. Gilbert – Regular Calculus and AP Calculus AB Instructors

Regular Calculus and AP Calculus AB Summer 2017 Assignment

As you begin Honors Calculus or AP Calculus, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand Calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. I assume you have the basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring should be easy for you by now. If not, you would not be going onto Honors Calculus or AP Calculus. There are several topics that students always need to review. There is a lot of material to cover before the AP exam in May and thus you will need to review these topics on your own this summer. This summer assignment is designed to help you review/relearn those topics that you need refreshed. I have attached some resources here and provided a few links to refer to if you need help. You can also email Mr. Modenou at kodjovi_modenou@dekalbschoolsga.org or Mr. Gilbert at martin_gilbert@dekalbschoolsga.org anytime this summer if you have questions.

I cannot stress enough how important it is to make sure you understand each concept covered on this summer assignment. Take your time to relearn algebraic and trigonometric topics you may have forgotten. If you do not feel confident with your answers to these questions or you need any guidance, please study your notes from prior math classes, reference the websites below, or contact me. Please don't "fake" your way through these problems. If you do not fully understand the packet, it will make your Calculus experience much more difficult. I do not want you to do poorly in Calculus because of a lack of prior knowledge.

Please do not wait until the last minute to complete the summer assignment. There are a lot of problems to solve and I want to make you take your time understanding each of them. At the same time, please do not finish the whole packet in the beginning of the summer. The point of this assignment is to refresh your memory of skills that are needed in the course and if you do all the work early, you may forget again before school starts.

The packet is due on the first day of school, no exceptions. If you forget to bring it with you, I will consider you unprepared for class and it will negatively affect your grade. The entire packet will be considered a project grade which accounts for 25% of your grade. There will be an in-class **test** on the material from the assignment on the 1st or 2nd week of school. I expect your work to be shown neatly on separate sheet of paper (graphing paper is preferred for the graphs) and answers clearly labeled. If I cannot find each question and answer, I will assume they are not given and you will receive no credit for the problem. Since I will not be able to monitor how you complete your packet, it is fine if you use a calculator. However, keep in mind that half of the AP exam does not allow a calculator to be used, so don't rely too heavily on one.

As I stated before, you may email me with any questions regarding the summer assignment. I look forward to working with each of you in the fall. Good luck and have a great summer!

Helpful Websites:

Algebra:
http://www.purplemath.com/modules/index.htm

Trigonometry:
http://math.com/homeworkhelp/Trigonometry.html

AP CALCULUS AB / REGULAR CALCULUS SUMMER ASSIGNMENT 2017

Teacher Name: _____ Student Name: _____ Period____

I. Basic Algebraic Rules

1. Are the following statements true? If not, change them to make them true.

a. $\frac{2k}{2k+4}=\frac{k}{k+4}$

b. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$

c. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$

d. $3\left(\frac{a}{b}\right)=\frac{3a}{3b}$

e. $3\left(\frac{a}{b}\right)=\frac{3a}{b}$

f. $3\left(\frac{a+b}{c}\right)=\frac{3a+b}{c}$

II. Complex Fractions & Rational Expressions

2. Simplify.

a. $\dfrac{\frac{x}{2}}{\frac{x}{4}}$

b. $h\div\frac{x+h}{h}$

c. $\dfrac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2}$

3. Write as a single fraction with the denominator in <u>factored form</u>.

a. $\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}$

b. $20\left(\frac{2}{x+1}-\frac{3}{x}\right)$

c. $x(1-2x)^{-\frac{3}{2}}+(1-2x)^{-\frac{1}{2}}$

d. $(3x-2)^{\frac{1}{2}}+x(3x-2)^{-\frac{1}{2}}$

d. $\dfrac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

III. Negative and Fractional Exponents

4. Simplify using only positive exponents. Do not rationalize the denominators.

a. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

b. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

c. $\left(\frac{x^{-2}}{y^{-1}}-x\right)^{-3}$

IV. Solving Equations and Factoring

5. Solve for y′ in *simplest form*.

a. $xy' + y = 1 + y'$ b. $3y^2y' + 2yy' = 5y' + 2x$ c. $3x^2yy' + 2xy^2 = 2yy'$

6. Solve the quadratic equation. Use any means from algebra: factoring, quadratic formula, graphing. Be sure answers are simplified.

a. $4x^2-21x-18=0$ b. $2x^2-3x+3=0$ c. $x^4-9x^2+8=0$

7. Factor completely (There should be no fractional or negative exponents.)

a. $3x^3+192$ b. $9x^2-3x-2$ c. $2\sqrt{x}-6x^{\frac{3}{2}}$

d. $\sin x + \tan x$ e. $e^{-x}-xe^{-x}+2x^2e^{-x}$ f. $2x^4 + 5x^3 - 3x^2$

V. Equations of lines

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

9. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1, 2).

10. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

12. Graph the equation $y = x^3 - x$ and answer the following questions.

a. Is the point (3, 2) on the graph?

b. Is the point (2, 6) on the graph?

c. Is the function odd, even or neither?

d. Find the x and y – intercept(s).

VI. Asymptotes and Intercepts

13. Find all intercepts and asymptotes.

a. $y^2 = x^2 - 4x$

b. $y = \dfrac{x^2 + 3x}{(3x+1)^2}$

c. $y = \dfrac{x^2 - 4}{x^2 - x - 12}$

d. $y = \dfrac{3x-1}{2x^2+x-6}$

VII. Domain

14. Use interval notation to identify the domain for each of the following functions.

a. $h(x) = \dfrac{1}{4x^2 - 21x - 18}$

b. $k(x) = \sqrt{x^2 - 5x - 14}$

c. $\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

d. $d(x) = \ln(2x - 12)$

VIII. Graphing Functions

15. Graph the function.

a. f(x) = $\begin{cases} 1 & x \le 0 \\ -1 & x > 0 \end{cases}$

b. f(x) = $\begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & [2, \infty) \end{cases}$

c. f(x) = $\sqrt{16 - x^2}$

16. Graph each.

a. $y=\begin{cases}3-x & x\le 1\\ 2x & x>1\end{cases}$ b. $y=\begin{cases}4-x^2 & x<1\\ \frac{3}{2}x+\frac{3}{2} & 1\le x\le 3\\ x+2 & x>3\end{cases}$ c. $y=\frac{|x+1|}{x+1}$ d. $y=|x-2|+3$

IX. Functions

17. Find f(1) – f(5) given f(x) = |x – 3| - 5.

18. Find f(x + 2) – f(2) given f(x) = x^2-3x+4.

19. Find f(x + h) for f(x) = x^2-2x-3.

20. Find $\frac{f(x+h)-f(x)}{h}$ if f(x) = $8x^2+1$.

21. Given f(x) = x – 3 and g(x) = $\sqrt{x}$, complete the following.

a. f(g(x)) = b. g(f(x)) = c. f(f(x)) =

22. Given $f(x)=\frac{1}{x-5}$ and g(x) = x^2-5, complete the following.

a. f(g(7)) = b. g(f(v)) = c. g(g(x)) =

X. Binomial Theorem

23. Use the Binomial Theorem to expand and simplify the following expressions.

a. $(x+5)^5$ b. $(3a-4b)^4$ c. $(2w+3)^6$

XI. Factor Theorem

24. Use the p over q method and synthetic division to factor the polynomial P(x). Then solve P(x)=0.

a. $P(x) = x^3 + 5x^2 - 2x - 24$ b. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

XII. Logarithms

25. Condense the expression $2\ln(x-3) + \ln(x+2) - 6\ln x$

26. Express y in terms of x.

a. lny = x + 2 b. lny = 2lnx + ln10 \

c. lny = 4lnx + 3 d. $x = \ln \dfrac{e^{x^2}}{4y}$

27. Solve for x.

a. $\ln e^3 = x$ b. $\ln e^x = 4$ c. lnx + lnx = 0 d. $e^{\ln 5} = x$

e. ln1 – lne = x f. ln6 + lnx – ln2 = 3 g. ln(x + 5) = ln(x – 1) – ln(x +1)

XIII. Trigonometry

28. Evaluate (without a calculator!!). NO decimals.

a. cos 0 b. sin 0 c. tan $\dfrac{\pi}{2}$ d. cos $\dfrac{\pi}{4}$ e. sin $\dfrac{\pi}{2}$ f. sin π

g. $\sin^{-1} \dfrac{\sqrt{3}}{2}$ h. $\tan^{-1} 1$ i. $\cos^{-1} \dfrac{1}{2}$ j. $\sec^{-1} \sqrt{2}$

k. If $\cos\theta = \dfrac{5}{13}$ and θ is in Quadrant II, Find the all the remaining trig functions.

29. Which of the following expressions are identical?

a. $\cos^2 x$ b. $(\cos x)^2$ c. $\cos x^2$

30. Which of the following expressions are identical?

a. $(\sin x)^{-1}$ b. arcsinx c. $\sin x^{-1}$ d. $\dfrac{1}{\sin x}$

31. Solve the following for the indicated variable on the interval $[0, 2\pi)$.

a. $3\cos x - 1 = 2$ b. $2\sin(2x) - \sqrt{3} = 0$ c. $\tan^2 x - 1 = 0$ d. $2\sin^2 x + \sin x = 1$

32. Complete the following trig identities

a. $\sin^2 x + \cos^2 x =$ b. $\tan^2 x + 1 =$ c. $\dfrac{1 - (\sin x + \cos x)^2}{2\sin x}$

XIV. Word Problems

33. Find the surface area of a box of height *h* whose base dimensions are *p* and *q* and satisfies the following conditons.

a. The box is closed.

b. The box has an open top.

c. The box has an open top and a square base with side length *p*.

34. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

35. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

36. A police car receives a radio call to catch a vehicle which is speeding down the highway at 80 mph. The police car, which is 12 miles away, drives after it at 108 mph. How long will it take for the police car to catch up?

37. The base of a triangle is 6 cm more than the height. If the area of the triangle is 140 square cm, what is the length of the base?

38. Two trains, the Express and the Commuter, leave the same station at the same time. The Express, which heads north, travels 10 km per hour faster than the Commuter, which goes east. If the trains are 100 km apart after 2 hours, find the speed of each train.

39. The depth, d, of a buoyant object t seconds after plunging into water can be found using the equation $d = -6t^2 + rt$, where r is the velocity at which the object strikes the water. If the object strikes the water at a velocity of 240 feet per second, find the maximum depth the rocket will reach and at what time. When will the rocket surface again?

Find the average rate of change for the following functions on the indicated intervals.

40. $f(x) = x^3 - 2x; [0, 4]$

41. $f(x) = 3\sqrt{x}; [4, 25]$

42. A car travels 420 miles over a period of 210 minutes. Find the average velocity of the car in miles per hour over this time period.

43. On January 1st 2003, the value of a stock was $135 per share. By December 1st 2003, the value of the stock had fallen to $38 per share. What is the average rate of change in the value of the stock in dollars per month?

44. In 1984, the Fizzy Cola company sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in the number of gallons of soda sold per year?

45. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?