Advanced Algebra/Algebra II
Summer Assignment

Instructor:_____   Name:

Results: _____   Class:

INSTRUCTION

Before begin your summer assignment, please take the time to visit the following websites to get familiar with each concept. Then, answer all 45 items. Please show all work for both constructed and selected response. If you do not show you work for an item, you will not get credit for this particular question.

http://www.montereyinstitute.org/courses/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U16_L1_T3_text_final.html

- Rational Exponents

http://www.sosmath.com/algebra/algebra.html

- Complex Numbers
- Quadratic Equations
- Factorization and Roots of Polynomials
- Solving Equations
- Systems of Equations
- Inequalities
- Logarithms and Exponential Functions

https://www.mathsisfun.com/algebra/binomial-theorem.html

- Binomial Theorem

http://www.shelovesmath.com/algebra/advanced-algebra/piecewise-functions

- Graphing Piecewise functions

https://www.mathsisfun.com/data/

- Mean, Median, Mode & Range
- Quartiles and interquartile Range
- Mean Deviation
- Standard Deviation
- Z-score
- Normal distribution

1. Consider the set of real numbers and the set of complex numbers.

    a. Is every real number also a complex number? Explain.

    b. Is every complex number also a real number? Explain.

    c. Which Venn diagram below accurately represents the two sets of numbers? Explain.

2. Simplify $\left(\sqrt[7]{z^3}\right)^8$. Assume z is positive.

  Ⓐ $z^{\frac{56}{3}}$

  Ⓑ $z^{\frac{24}{7}}$

  Ⓒ $z^{\frac{24}{56}}$

  Ⓓ $z^{\frac{11}{15}}$

3. Write $\left(\sqrt{uv^3}\right)^5$ using rational exponents. Assume u and v are both positive.

  Ⓐ $u^{\frac{5}{2}}v^{\frac{15}{2}}$

  Ⓒ $u^{\frac{2}{5}}v^{\frac{2}{15}}$

  Ⓑ u^5v^{15}

  Ⓓ $u^{\frac{11}{2}}v^{\frac{13}{2}}$

4. Show that $\left(a^{-\frac{1}{m}}\right)^{-\frac{1}{n}} = \sqrt[mn]{a}$ for positive values of m, n, and a. Then use this information to simplify

$\left[\left(jk^4\right)^{-\frac{1}{5}}\right]^{-\frac{1}{3}}$ for positive values of j and k. Show all work.

5. Which of the following is equivalent to $\sqrt{-121}$?

(A) 11

(B) -11

(C) $11i$

(D) $121i$

6. Find all solutions of the equation. $x^2 = -484$.

(A) $x = 22$

(B) $x = -22$

(C) $x = 22i$

(D) $x = -22i$

7. Simplify each expression and tell whether it represents a real number or a non-real number.

   a. $\sqrt{144} - \sqrt{64}$

   b. $\sqrt{144} + \sqrt{-64}$

8. How many real solutions and how many non-real solutions does the equation $x^2 + 5 = 0$ have? Describe what implications your answer has for graphing the function $y = x^2 + 5$.

9. Why is it imprecise to say that the equation $25x^2 + 27 = 0$ has no solution? Find all solutions of the equation.

10. Which of these expressions is equal to $4 - 7i$?

(A) $(6 - i) - (2 - 8i)$

(B) $(6 + i) - (2 + 8i)$

(C) $(6 + i) + (2 + 8i)$

(D) $(6 - i) + (2 - 8i)$

11. Which of the following sums, differences, and products can be simplified to $6 - 3i$?

$\textbf{A}$ $(9 - 5i) + (3 - 2i)$

$\textbf{B}$ $(4 + 2i) + (2 - 5i)$

$\textbf{C}$ $(9 - 5i) - (3 - 2i)$

$\textbf{D}$ $(4 + 2i) - (2 - 5i)$

$\textbf{E}$ $3i(-1 + 2i)$

$\textbf{F}$ $3i(-1 - 2i)$

12. Find the Product

  a. $(3 + 4i)(3 + 4i)$.

  b. $(3 + 4i)(3 - 4i)$.

13. Complete the table of values. Describe the pattern you see. Then use the pattern to find the values of i^9, i^{26}, and i^{100}. Explain your reasoning.

i	i^2	i^3	i^4	i^5	i^6	i^7	i^8
$\sqrt{-1}$							

14. When $x^3 - 125$ is written as a product of a binomial and a trinomial, what is the trinomial factor?

$\textbf{A}$ $x^2 - 5x + 25$
$\textbf{B}$ $x^2 + 5x + 25$
$\textbf{C}$ $x^2 - 10x + 25$
$\textbf{D}$ $x^2 + 10x + 25$

15. Do the equations $x^2 - 4x + 21 = 0$ and $x^2 + 4x - 21 = 0$ have the same type (real or non-real) and number of solutions? Describe the solutions of each equation, and then solve each equation over the set of complex numbers.

16. Consider the equation $4x^2 + 8x + c = 0$.

    a. Describe all values of c such that the equation $4x^2 + 8x + c = 0$ has two non-real solutions. Explain.

    b. Solve the equation $4x^2 + 8x + 13 = 0$ over the set of complex numbers.

17. Write $x^6 - 729$ in factored form. Show your work.

18. What is the coefficient of the x^4-term in the expanded form of $(2x - 7)^6$?

    (A) -11,760
    (B) -784
    (C) 784
    (D) 11,760

19. Use the binomial theorem to write a polynomial identity for $(a - b)^5$. Show your work.

20. Which equation has an extraneous solution that is negative?

Ⓐ $\sqrt{5x + 29} = x + 7$

Ⓑ $\dfrac{16}{x + 10} = \dfrac{2x + 36}{x + 10}$

Ⓒ $\sqrt{7x - 5} + 5 = x$
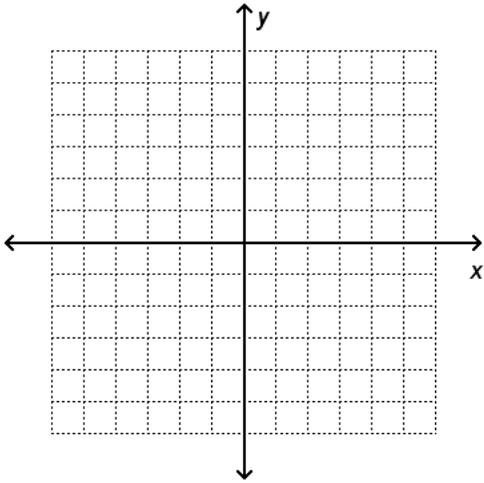
Ⓓ $\dfrac{3}{x - 8} = \dfrac{5x - 34}{x^2 - 14x + 48}$

21. Solve the radical equation $\sqrt{2x + 1} + 7 = x.$ Show your work. Remember to check for extraneous solutions.

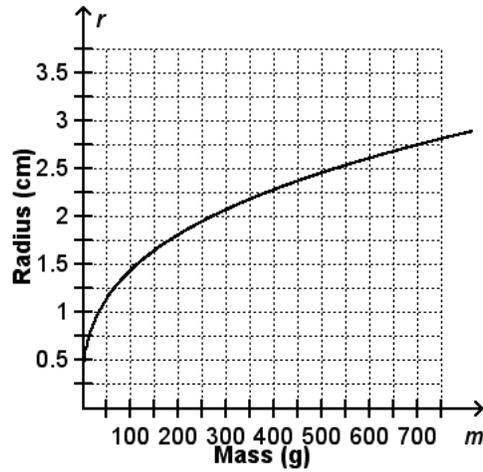22. Given the following system of equations, solve for x, y, and z. Show your work. $\begin{cases} 2x + 3y + z = 27 \\ x + 5y - 3z = 52 \\ -3x + y + 2z = -16 \end{cases}$

23. Graph the following system of equations and then solve the system graphically.

$\begin{cases} 2x + y = -12 \\ y = x^2 - 5 \end{cases}$

24. The graph shows the radius *r* of a ball bearing with uniform density as a function of its mass *m*. What type of function best describes the graph?

Ⓐ Quadratic function

Ⓑ Cubic function

Ⓒ Cube root function

Ⓓ Square root function

25. Suppose a sheet of paper is repeatedly folded in half. The function $A(n) = A_0 \left(\dfrac{1}{2} \right)^n$ models the area of

the sheet of paper after n folds. What is the A-intercept of the graph of $A(n)$?
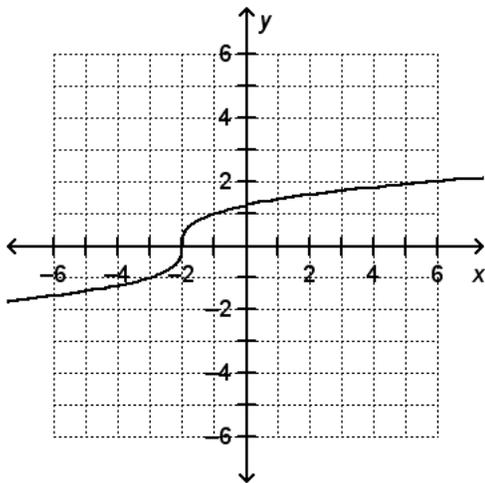
(A) 0   (C) n

(B) $\dfrac{1}{2}$ (D) A_0

26. Graph the piecewise-defined function $f(x)$ given below. Then describe the function's domain and range and any intercepts that the graph has.

$$f(x) = \begin{cases} -2x - 2 & \text{if } x < 2 \\ 2 & \text{if } 2 \le x \le 5 \\ 5\sqrt{x-5} + 2 & \text{if } x > 5 \end{cases}$$
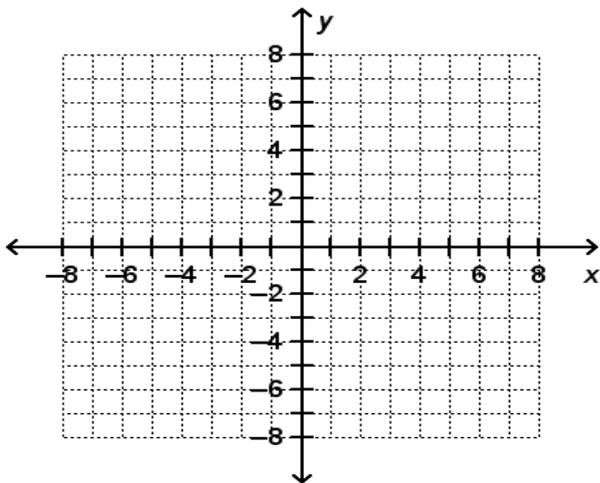
27. The graph of which function is shown?

(A) $y = \sqrt{x+2}$

(B) $y = \sqrt{x-2}$

(C) $y = \sqrt[3]{x+2}$

(D) $y = \sqrt[3]{x-2}$

28. Consider the function $f(x) = 2x^2 + 4x - 30$. Classify each statement.

a. The vertex of the graph of $f(x)$ is $(1, -32)$.　　　　○ True　　○ False

b. The zeros of $f(x)$ are $x = 3$ and $x = -5$.　　　　○ True　　○ False

c. The graph of $f(x)$ opens down.　　　　○ True　　○ False

d. The axis of symmetry is $x = -1$.　　　　○ True　　○ False

e. The y-intercept of $f(x)$ is -30.　　　　○ True　　○ False

29. The population of city A was 12,831 in 1900 and grew 5% per year for the next 10 years. For the same period, the function $P(t) = 11{,}572(1.07)t$ models the population of city B, where P is the population and t is the time, in years since 1900.

What was the population of each city in 1900? Which city had the greater population?

At what annual rate was the population of each city increasing over this period? The population of which city was growing faster?

What was the population of each city in 1910? Which city had the greater population?

30. Let $f(x) = 15(4)^{x-2} + 7$. Find the inverse of $f(x)$ and use it to find x such that $f(x) = 247$. Show your work.

31. Identify and correct the error that a student made when solving the equation $7(10)^{0.25x} = 5375$.

$$7(10)^{0.25x} = 5375$$
$$10^{0.25x} = \frac{5375}{7}$$
$$\log 10^{0.25x} = \log \frac{5375}{7}$$
$$0.25x \log 10 = \log \frac{5375}{7}$$
$$0.25x = \log 5375 - \log 7$$
$$0.25x = \log 5368$$
$$x = 4 \log 5368$$
$$x \approx 14.92$$

32. Find the inverse of $f(x) = \sqrt{x-2}$. Show your work. Also, describe how the domain and range of $f(x)$ compare to the domain and range of its inverse. What are the domain and range of the inverse?

33. The function $P(t) = 50(2)^{0.25t}$ models the population P of a certain type of bacteria after t hours. How long does it take the initial population to double?

(A) 0.25 hour

(B) 2 hours

(C) 4 hours

(D) 50 hours

34. The function $GDP(t) = 0.504(1.18)^t$ models the gross domestic product GDP, in trillions of dollars, as a function of the time t, in years, of a nation from 2002 to 2011. What is the annual percentage increase in the gross domestic product during this period?

(A) 0.18%
(B) 0.504%
(C) 1.18%
(D) 18%

35. Consider the function $f(x) = 4x^2 + 4x - 15$.

    a. Factor the expression $4x^2 + 4x - 15$. What are the zeros of $f(x)$?

    b. What are the coordinates of the vertex of $f(x)$? Is the vertex the maximum or minimum value of the function? Explain.

36. Which of these functions describe exponential growth?

  Ⓐ $f(t) = 1.25^t$

  Ⓑ $f(t) = 2(0.93)^{0.5t}$

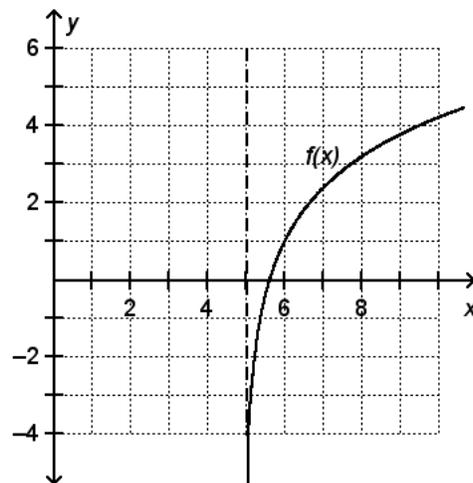 Ⓒ $f(t) = 3(1.07)^{3t}$

  Ⓓ $f(t) = 18(0.85)^t$

  Ⓔ $f(t) = 0.5(1.05)^t$

  Ⓕ $f(t) = 3(1.71)^{5t}$

  Ⓖ $f(t) = 0.68^{2t}$

  Ⓗ $f(t) = 8(1.56)^{1.4t}$

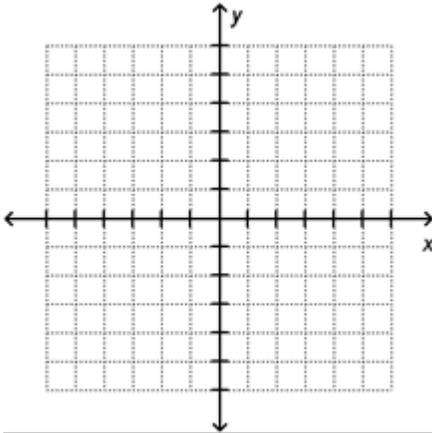37. The graph of a logarithmic function is shown. What is the domain of the function?

  Ⓐ $x \geq 5$ Ⓒ $-\infty < y < \infty$
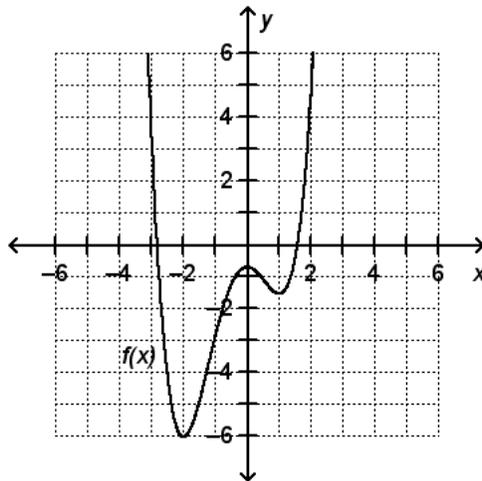
  Ⓑ $x > 5$ Ⓓ $y < 5$

38. Identify the zeros of the polynomial function $p(x) = -x^3 - x^2 + 16x + 16$, and describe the function's end behavior. Then graph the function using the zeros and the end behavior and plotting any additional points as needed.

39. The graph of a fourth-degree polynomial function $f(x)$ is shown. Use the graph to determine the domain and range of $f(x)$.
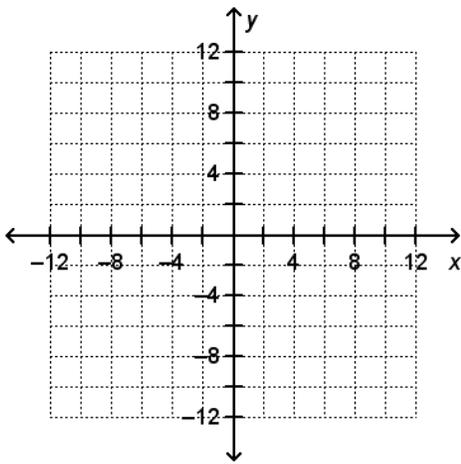

Ⓐ Domain: $-\infty < x < \infty$; range: $f(x) \geq -6$

Ⓑ Domain: $-\infty < x < \infty$;
   range: $-\infty < f(x) < \infty$

Ⓒ Domain: $-4 < x < 2$; range: $f(x) \geq -6$

Ⓓ Domain: $-4 < x < 2$;
   range: $-\infty < f(x) < \infty$

40. On which of the following intervals is the average rate of change of the function $f(x) = x^3 - 4x$ the greatest?

(A) From $x = -3$ to $x = -1$

(B) From $x = -1$ to $x = 1$

(C) From $x = 1$ to $x = 3$

(D) From $x = 3$ to $x = 5$

41. Consider the function $y = 2\sqrt{x+2} + 4.$

   a. Graph the function.

   b. What is the domain and the range of the function?

42. The wing lengths of houseflies are normally distributed with a mean of 45.5 mm and a standard deviation of 3.92 mm. Use the standard normal distribution to estimate each percent.

a. The percent of houseflies with wing lengths over 35 millimeters.

b. The percent of houseflies with wing lengths over 50 millimeters.

43. The scores on a recent district-wide math test are normally distributed with a mean of 82 points and a standard deviation of 5 points. Use the standard normal distribution to answer each question.

a. What percent of students scored between 70 and 75 on the test? Show your work.

b. What percent of students scored at least 90 on the test? Show your work.

c. What percent of students scored at most 65 on the test? Show your work.

44. The heights of the male students at Bart's school are normally distributed with a mean of 68 inches and a standard deviation of 2 inches.

a. What percent of the male students at Bart's school are more than 68 inches tall? Explain.

b. What percent of the male students at Bart's school are less than 64 inches tall? Explain. (Hint: Use the 68-95-99.7 rule.)

45. The scores for the mathematics portion of a standardized test are normally distributed with a mean of 514 points and a standard deviation of 117 points. What is the probability that a randomly selected student has a score of 610 points or less on the test? Use the standard normal distribution to estimate the probability.

A. 29.4%
B. 79.4%
C. 20.6%
D. 68%