Rates, Ratios, and Proportions

Warm Up

Solve each equation. Check your answer.

1. $6x = 36$ 6

2. $\frac{x}{4} = 12$ 48

3. $5m = 18$ 3.6

4. $\frac{r}{21} = -3$ −63

5. $8y = 18.4$ 2.3

Multiply.

6. $8\left(\frac{7}{8}\right)$ 7

7. $12\left(\frac{5}{6}\right)$ 10

Objectives

Write and use ratios, rates, and unit rates.

Write and solve proportions.

Vocabulary

ratio	proportion
rate	cross products
scale	scale drawing
unit rate	scale model
conversion factor	dimensional analysis

Rates, Ratios, and Proportions

A **ratio** is a comparison of two quantities by division. The ratio of *a* to *b* can be written *a:b* or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be *equivalent.*

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion.**

Reading Math

Read the proportion $\frac{1}{15} = \frac{x}{675}$ as

"1 is to 15 as x is to 675".

Example 1: Using Ratios

The ratio of the number of bones in a human's ears to the number of bones in the skull is 3:11. There are 22 bones in the skull. How many bones are in the ears?

$$\frac{\text{ears} \rightarrow 3}{\text{skull} \rightarrow 11}$$

Write a ratio comparing bones in ears to bones in skull.

$$\frac{3}{11} = \frac{x}{22}$$

Write a proportion. Let x be the number of bones in ears.

$$22\left(\frac{x}{22}\right) = 22\left(\frac{3}{11}\right)$$

Since x is divided by 22, multiply both sides of the equation by 22.

$$x = 6$$

There are 6 bones in the ears.

Check It Out! Example 1

The ratio of games won to games lost for a baseball team is 3:2. The team has won 18 games. How many games did the team lose?

$$\frac{\text{won} \rightarrow 3}{\text{lost} \rightarrow 2}$$

Write a ratio comparing games lost to games won.

$$\frac{3}{2} = \frac{18}{x}$$

Write a proportion. Let x be the number of games lost.

$$x\left(\frac{3}{2}\right) = x\left(\frac{18}{x}\right)$$

Since 18 is divided by x, multiply both sides of the equation by x.

$$\frac{3}{2}x = 18$$

Check It Out! Example 1 Continued

$$\frac{3}{2}x = 18$$

$$\frac{2}{3}\left(\frac{3}{2}x\right) = \frac{2}{3}(18)$$

Since x is multiplied by $\frac{3}{2}$, multiply both sides of the equation by $\frac{2}{3}$.

$$x = 12$$

The team lost 12 games.

Rates, Ratios, and Proportions

A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as *unit rates.* A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal. You can convert any rate to a unit rate.

Example 2: Finding Unit Rates

Raulf Laue of Germany flipped a pancake 416 times in 120 seconds to set the world record. Find the unit rate. Round your answer to the nearest hundredth.

$\frac{416}{120} = \frac{x}{1}$ *Write a proportion to find an equivalent ratio with a second quantity of 1.*

$3.47 \approx x$ *Divide on the left side to find x.*

The unit rate is about 3.47 flips/s.

Check It Out! Example 2

Cory earns $52.50 in 7 hours. Find the unit rate.

$\frac{52.50}{7} = \frac{x}{1}$ *Write a proportion to find an equivalent ratio with a second quantity of 1.*

$7.5 = x$ *Divide on the left side to find x.*

The unit rate is $7.50.

Rates, Ratios, and Proportions

Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft.}}$, in which the two quantities are equal but use different units, is called a **conversion factor**. To convert a rate from one set of units to another, multiply by a conversion factor.

Example 3A: Using Dimensional Analysis

A fast sprinter can run 100 yards in approximately 10 seconds. Use dimensional analysis to convert 100 yards to miles. Round to the nearest hundredth. (*Hint:* There are 1760 yards in a mile.)

$$100 \text{ yd} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}}$$

Multiply by a conversion factor whose first quantity is yards and whose second quantity is miles.

$$\approx 0.06$$

100 yards is about 0.06 miles.

Helpful Hint

In Additional Example 3A , “yd” appears to divide out, leaving “mi,” as the unit. Use this strategy of “dividing out” units when using dimensional analysis.

Example 3B: Using Dimensional Analysis

A cheetah can run at a rate of 60 miles per hour in short bursts. What is this speed in feet per minute?

Step 1 Convert the speed to feet per minute.

$$\frac{60 \text{ mi}}{1 \text{ h}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{\text{h}}{\text{min}}$$

$$\frac{5280 \text{ ft}}{1 \text{ min}} \cdot \frac{\text{ft}}{\text{min}}$$

To convert the first quantity in a rate, multiply by a conversion factor with that unit in the first quantity.

The speed is 316,800 feet per minute.

Example 3B: Using Dimensional Analysis Continued

The speed is 5280 feet per minute.

Check that the answer is reasonable.

- There are 60 min in 1 h, so 5280 ft/min is 60(5280) = 316,800 ft/h.
- There are 5280 ft in 1 mi, so 316,800 ft/h is $\frac{316,800}{5280} = 60$ mi/h. This is the given rate in the problem.

Check It Out! Example 3

A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second? Round your answer to the nearest tenth, and show that your answer is reasonable.

Use the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$ to convert miles to feet and use the conversion factor $\frac{1\text{ h}}{3600\text{ s}}$ to convert hours to seconds.

$$\frac{56\text{ mi}}{4\text{ h}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{3600\text{ s}} \approx \frac{20.5\text{ ft}}{1\text{ s}}$$

The speed is about 20.5 feet per second.

Check It Out! Example 3 Continued

Check that the answer is reasonable. The answer is about 20 feet per second.

- There are 60 seconds in a minute so 60(20) = 1200 feet in a minute.
- There are 60 minutes in an hour so 60(1200) = 72,000 feet in an hour.
- Since there are 5,280 feet in a mile 72,000 ÷ 5,280 = about 14 miles in an hour.
- The cyclist rode for 4 hours so 4(14) = about 56 miles which is the original distance traveled.

Rates, Ratios, and Proportions

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ *and* $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products property.

Cross Products Property

WORDS	NUMBERS	ALGEBRA
In a proportion, cross products are equal.	$\frac{2}{3} = \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$	If $\frac{a}{b} = \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$ then $ad = bc$.

Example 4: Solving Proportions

Solve each proportion.

A. $\frac{3}{9} = \frac{5}{m}$

$\frac{3}{9} \times \frac{5}{m}$ *Use cross products.*

$3(m) = 5(9)$

$3m = 45$

$\frac{3m}{3} = \frac{45}{3}$ *Divide both sides by 3.*

$m = 15$

B. $\frac{6}{y-3} = \frac{2}{7}$

$\frac{6}{y-3} \times \frac{2}{7}$ *Use cross products.*

$6(7) = 2(y - 3)$

$42 = 2y - 6$

$+6 \qquad +6$

$48 = 2y$ *Add 6 to both sides.*

$\frac{48}{2} = \frac{2y}{2}$ *Divide both sides by 2.*

$24 = y$

Check It Out! Example 4

Solve each proportion.

A. $\frac{-5}{2} = \frac{y}{8}$

$\frac{-5}{2} \times \frac{y}{8}$ *Use cross products.*

$2(y) = -5(8)$

$2y = -40$

$\frac{2y}{2} = \frac{-40}{2}$ *Divide both sides by 2.*

$y = -20$

B. $\frac{g+3}{5} = \frac{7}{4}$

$\frac{g+3}{5} \times \frac{7}{4}$ *Use cross products.*

$4(g+3) = 5(7)$

$4g + 12 = 35$

Subtract 12 from both sides.

$-12 \quad -12$

$4g = 23$

$\frac{4g}{4} = \frac{23}{4}$ *Divide both sides by 4.*

$g = 5.75$

Rates, Ratios, and Proportions

A **scale** is a ratio between two sets of measurements, such as 1 in:5 mi. A **scale drawing** or **scale model** uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

Example 5A: Scale Drawings and Scale Models

A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft.

A wall on the blueprint is 6.5 inches long. How long is the actual wall?

$\frac{\text{blueprint} \longrightarrow 1 \text{ in.}}{\text{actual} \longrightarrow 3 \text{ ft.}}$ *Write the scale as a fraction.*

$\frac{1}{3} = \frac{6.5}{x}$ *Let x be the actual length.*

$x \cdot 1 = 3(6.5)$ *Use the cross products to solve.*

$x = 19.5$

The actual length of the wall is 19.5 feet.

Example 5B: Scale Drawings and Scale Models

A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft.

One wall of the house will be 12 feet long when it is built. How long is the wall on the blueprint?

$$\frac{\text{blueprint} \longrightarrow 1 \text{ in.}}{\text{actual} \longrightarrow 3 \text{ ft.}}$$

Write the scale as a fraction.

$$\frac{1}{3} = \frac{x}{12}$$

Let x be the actual length.

$$12 = 3x$$

Use the cross products to solve.

$$\frac{12}{3} = \frac{3x}{3}$$

Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

$$4 = x$$

The wall on the blueprint is 4 inches long.

Check It Out! Example 5

A scale model of a human heart is 16 ft. long. The scale is 32:1. How many inches long is the actual heart it represents?

$$\frac{\text{model}}{\text{actual}} \rightarrow \frac{32 \text{ in.}}{1 \text{ in.}}$$

Write the scale as a fraction.

Convert 16 ft to inches.
Let x be the actual length.

$$\frac{32}{1} = \frac{192}{x}$$

$$32x = 192$$

Use the cross products to solve.

$$\frac{32x}{32} = \frac{192}{32}$$

Since x is multiplied by 32, divide both sides by 32 to undo the multiplication.

$$x = 6$$

The actual heart is 6 inches long.

Lesson Quiz: Part 1

1. In a school, the ratio of boys to girls is 4:3. There are 216 boys. How many girls are there?

162

2. Nuts cost $10.75 for 3 pounds. Find the unit rate in dollars per pound.

$3.58/lb

3. Sue washes 25 cars in 5 hours. Find the unit rate in cars per hour.

5 cars/h

4. A car travels 180 miles in 4 hours. Use dimensional analysis to convert the car's speed to feet per minute?

3960 ft/min

Lesson Quiz: Part 2

Solve each proportion.

5. $\frac{8}{12} = \frac{g}{9}$ 6

6. $\frac{3}{z-4} = \frac{2}{8}$ 16

7. A scale model of a car is 9 inches long. The scale is 1:18. How many inches long is the car it represents? 162 in.