

Warm Up

Lesson Presentation

Lesson Quiz

Holt McDougal Algebra 1

Warm Up

Solve each equation. Check your answer.

- **1.** 6x = 36 6
- **2.** $\frac{x}{4} = 12$ **48**
- **3.** 5*m* = 18 **3.**6

4.
$$\frac{r}{21} = -3 - 63$$

5. 8*y* = 18.4 2.3

Multiply.

7.
$$12\left(\frac{5}{6}\right)$$
 10

Holt McDougal Algebra 1

6. $8\left(\frac{7}{8}\right)$ **7**

Objectives

Write and use ratios, rates, and unit rates. Write and solve proportions.

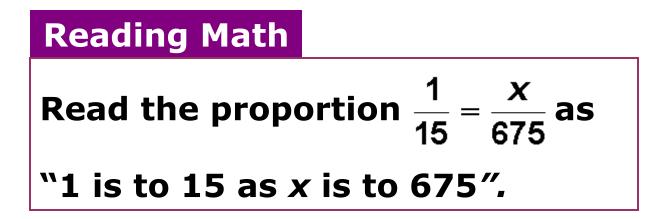
Holt McDougal Algebra 1

Vocabulary

ratioproportionratecross productsscalescale drawingunit ratescale modelconversiondimensionalfactoranalysis

A <u>ratio</u> is a comparison of two quantities by division. The ratio of *a* to *b* can be written *a*:*b* or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be *equivalent*.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.



Holt McDougal Algebra 1

Example 1: Using Ratios

The ratio of the number of bones in a human's ears to the number of bones in the skull is 3:11. There are 22 bones in the skull. How many bones are in the ears?

 $\frac{\text{ears} \rightarrow 3}{\text{skull} \rightarrow 11}$ $\frac{3}{11} = \frac{x}{22}$ $22\left(\frac{x}{22}\right) = 22\left(\frac{3}{11}\right)$ x = 6

Write a ratio comparing bones in ears to bones in skull.

Write a proportion. Let x be the number of bones in ears.

Since x is divided by 22, multiply both sides of the equation by 22.

There are 6 bones in the ears.

Holt McDougal Algebra 1

Check It Out! Example 1

The ratio of games won to games lost for a baseball team is 3:2. The team has won 18 games. How many games did the team lose?

- $\frac{\text{won} \rightarrow 3}{\text{lost} \rightarrow 2}$ $\frac{3}{2} = \frac{18}{x}$ $x\left(\frac{3}{2}\right) = x\left(\frac{18}{x}\right)$ $\frac{3}{2}x = 18$
- Write a ratio comparing games lost to games won.
- Write a proportion. Let x be the number of games lost.

Since 18 is divided by x, multiply both sides of the equation by x.

Check It Out! Example 1 Continued

$$\frac{3}{2}x = 18$$

$$\frac{2}{3}\left(\frac{3}{2}x\right) = \frac{2}{3}(18)$$
 Since x is multiplied by $\frac{3}{2}$, multiply both sides of the equation by $\frac{2}{3}$.
$$x = 12$$

The team lost 12 games.

Holt McDougal Algebra 1

A <u>rate</u> is a ratio of two quantities with different units, such as $\frac{34 \text{ mi.}}{2 \text{ gal.}}$. Rates are usually written as *unit rates.* A <u>unit rate</u> is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi.}}{1 \text{ gal.}}$, or 17 mi/gal. You can convert any rate to a unit rate.

Example 2: Finding Unit Rates

Raulf Laue of Germany flipped a pancake 416 times in 120 seconds to set the world record. Find the unit rate. Round your answer to the nearest hundredth.

| | X | Write a proportion to find an equivalent |
|-----|---|--|
| 120 | 1 | ratio with a second quantity of 1. |

3.47 $\approx x$ Divide on the left side to find x.

The unit rate is about 3.47 flips/s.

Holt McDougal Algebra 1

Check It Out! Example 2

Cory earns \$52.50 in 7 hours. Find the unit rate.

- $\frac{52.50}{7} = \frac{x}{1}$ Write a proportion to find an equivalent ratio with a second quantity of 1.
 - 7.5 = x Divide on the left side to find x.

The unit rate is \$7.50.

Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft.}}$, in which the two quantities are equal but use uifferent units, is called a **conversion factor**. To convert a rate from one set of units to another, multiply by a conversion factor.

Example 3A: Using Dimensional Analysis

A fast sprinter can run 100 yards in approximately 10 seconds. Use dimensional analysis to convert 100 yards to miles. Round to the nearest hundredth. (*Hint:* There are 1760 yards in a mile.)

 $100 \text{ yd} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}}$ Multiply by a conversion factor whose
first quantity is yards and whose
second quantity is miles.

100 yards is about 0.06 miles.

Helpful Hint

In Additional Example 3A , "yd" appears to divide out, leaving "mi," as the unit. Use this strategy of "dividing out" units when using dimensional analysis.

Example 3B: Using Dimensional Analysis

A cheetah can run at a rate of 60 miles per hour in short bursts. What is this speed in feet per minute?

Step 2 Convert the speed to feet per house.

| 60 mi | 5280 f | th | To convert the first quantity in a | |
|-------------|--------|-----|--|--|
| 1 h | 1 mi | min | rate, multiply by a conversion | |
| 5280 ft) ft | | | factor with that unit in the siestond quantity. | |
| 1 min | | | | |

The speed is 52608 feet paintater.

Example 3B: Using Dimensional Analysis Continued

The speed is 5280 feet per minute.

Check that the answer is reasonable.

- There are 60 min in 1 h, so 5280 ft/min is 60(5280) = 316,800 ft/h.
- There are 5280 ft in 1 mi, so 316,800 ft/h is $\frac{316,800}{5280} = 60$ mi/h. This is the given rate in the problem.

Check It Out! Example 3

A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second? Round your answer to the nearest tenth, and show that your answer is reasonable.

Use the conversion factor $\frac{5280 \text{ ft}}{1 \text{ mi}}$ to convert miles to feet and use the conversion factor $\frac{1 \text{ h}}{3600 \text{ s}}$ to convert hours to seconds.

 $\frac{56 \text{ mi}}{4 \text{ h}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \approx \frac{20.5 \text{ ft}}{1 \text{ s}}$

The speed is about 20.5 feet per second.

Holt McDougal Algebra 1

Check It Out! Example 3 Continued

Check that the answer is reasonable. The answer is about 20 feet per second.

- There are 60 seconds in a minute so 60(20)
- = 1200 feet in a minute.
- There are 60 minutes in an hour so 60(1200)
- = 72,000 feet in an hour.
- Since there are 5,280 feet in a mile 72,000 \div 5,280 = about 14 miles in an hour.
- The cyclist rode for 4 hours so 4(14) = about56 miles which is the original distance traveled.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products property.

| Cross Products Property | | | | | | | |
|--|---|---|--|--|--|--|--|
| WORDS | NUMBERS | ALGEBRA | | | | | |
| In a proportion, cross products are equal. | $\frac{2}{3} \cdot \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$ | If $\frac{a}{b}$ and $b \neq 0$ and $d \neq 0$ then $ad = bc$. | | | | | |

Holt McDougal Algebra 1

Example 4: Solving Proportions

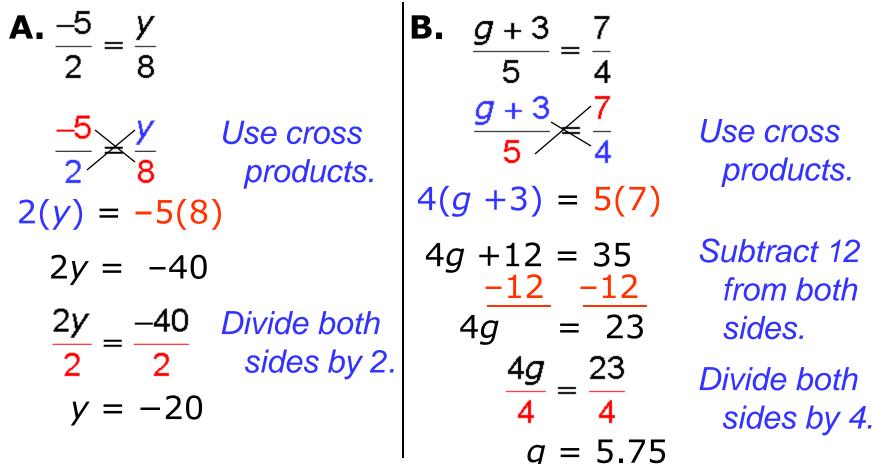
Solve each proportion.

| A. $\frac{3}{9} = \frac{5}{m}$ | B. $\frac{6}{y-3} = \frac{2}{7}$ | Use cross |
|---|---|----------------------------|
| $\overline{9}^{\times}\overline{m}$ pro | cross $\frac{6}{y-3}$ $\frac{2}{7}$ $\frac{7}{7}$ | products. |
| <mark>3(m)</mark> = 5(9) 3m = 45 | 6(7) = 2(y) 42 = 2y +6 | |
| $\frac{3m}{2} = \frac{45}{Divid}$ | $\begin{array}{c c} \hline +0\\ \hline 48 = 2y\\ \hline 48 = 2x\\ \hline 48 = $ | both sides. Divide both |
| m = 15 | $\begin{vmatrix} \frac{1}{2} \\ \frac{2}{24} = \frac{1}{2} \\ \frac{24}{24} = y$ | sides by 2. |

Holt McDougal Algebra 1

Check It Out! Example 4

Solve each proportion.



A <u>scale</u> is a ratio between two sets of measurements, such as 1 in:5 mi. A <u>scale drawing</u> or <u>scale model</u> uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

Example 5A: Scale Drawings and Scale Models

A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft.

A wall on the blueprint is 6.5 inches long. How long is the actual wall?

blueprint $\longrightarrow 1$ in. actual $\longrightarrow 3$ ft. Write the scale as a fraction.

- $\frac{1}{3} = \frac{6.5}{x}$ Let x be the actual length.
- $x \bullet 1 = 3(6.5)$ Use the cross products to solve.
- *x* = 19.5

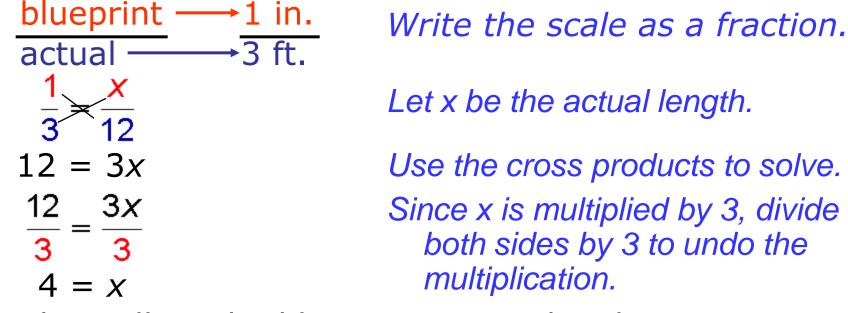
The actual length of the wall is 19.5 feet.

Holt McDougal Algebra 1

Example 5B: Scale Drawings and Scale Models

A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft.

One wall of the house will be 12 feet long when it is built. How long is the wall on the blueprint?



The wall on the blueprint is 4 inches long.

Holt McDougal Algebra 1

Check It Out! Example 5

A scale model of a human heart is 16 ft. long. The scale is 32:1. How many inches long is the actual heart it represents?

 $\frac{\text{model} \longrightarrow 32 \text{ in.}}{\text{actual} \longrightarrow 1 \text{ in.}}$ $\frac{32}{1} \xrightarrow{192}{x}$ 32x = 192 $\frac{32x}{32} = \frac{192}{32}$ x = 6

Write the scale as a fraction. Convert 16 ft to inches. Let x be the actual length.

Use the cross products to solve.

Since x is multiplied by 32, divide both sides by 32 to undo the multiplication.

The actual heart is 6 inches long.

Lesson Quiz: Part 1

- In a school, the ratio of boys to girls is 4:3.
 There are 216 boys. How many girls are there?
 162
- 2. Nuts cost \$10.75 for 3 pounds. Find the unit rate in dollars per pound.
 \$3.58/lb
- **3.** Sue washes 25 cars in 5 hours. Find the unit rate in cars per hour. 5 cars/h
- A car travels 180 miles in 4 hours. Use dimensional analysis to convert the car's speed to feet per minute?
 3960 ft/min

Lesson Quiz: Part 2

Solve each proportion.

5.
$$\frac{8}{12} = \frac{g}{9}$$
 6
6. $\frac{3}{z-4} = \frac{2}{8}$ **16**

7. A scale model of a car is 9 inches long. The scale is 1:18. How many inches long is the car it represents? 162 in.