Solving Systems by Elimination

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Simplify each expression.

1. $3x + 2y - 5x - 2y$ $-2x$

2. $5(x - y) + 2x + 5y$ $7x$

3. $4y + 6x - 3(y + 2x)$ y

4. $2y - 4x - 2(4y - 2x)$ $-6y$

Write the least common multiple.

5. 3 and 6 6

6. 4 and 10 20

7. 6 and 8 24

8. 2 and 5 10

Objectives

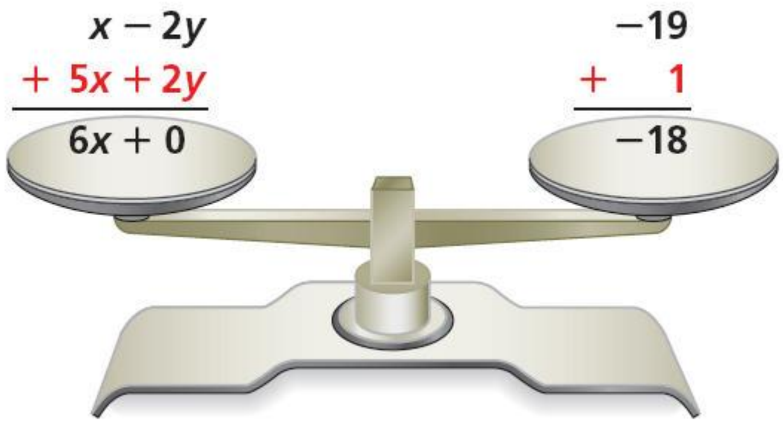
Solve systems of linear equations in two variables by elimination.

Compare and choose an appropriate method for solving systems of linear equations.

Another method for solving systems of equations is *elimination.* Like substitution, the goal of elimination is to get one equation that has only one variable.

Remember that an equation stays balanced if you add equal amounts to both sides.

Consider the system $\begin{cases} x - 2y = 19 \\ 5x + 2y = 1 \end{cases}$. Since $5x + 2y = 1$, you can add $5x + 2y$ to one side of the first equation and 1 to the other side and the balance is maintained.



Since $-2y$ and $2y$ have opposite coefficients, you can eliminate the y by adding the two equations. The result is one equation that has only one variable: $6x = -18$.

When you use the elimination method to solve a system of linear equations, align all like terms in the equations. Then determine whether any like terms can be eliminated because they have opposite coefficients.

Solving Systems of Equations by Elimination	
Step 1	Write the system so that like terms are aligned.
Step 2	Eliminate one of the variables and solve for the other variable.
Step 3	Substitute the value of the variable into one of the original equations and solve for the other variable.
Step 4	Write the answers from Steps 2 and 3 as an ordered pair, (x, y), and check.

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.

Example 1: Elimination Using Addition

Solve $\begin{cases} 3x - 4y = 10 \\ x + 4y = -2 \end{cases}$ **by elimination.**

Step 1

$$\begin{aligned} 3x - 4y &= 10 \\ x + 4y &= -2 \end{aligned}$$

Align like terms. −4y and +4y are opposites.

Step 2

$$4x + 0 = 8$$

Add the equations to eliminate y.

$$4x = 8$$

Simplify and solve for x.

$$\frac{4x}{4} = \frac{8}{4}$$

Divide both sides by 4.

$$x = 2$$

Example 1 Continued

Step 3 $x + 4y = -2$ — *Write one of the original equations.*

$2 + 4y = -2$ — *Substitute 2 for x.*

$\underline{-2} \qquad \underline{-2}$ — *Subtract 2 from both sides.*

$4y = -4$

$\frac{4y}{4} = \frac{-4}{4}$ — *Divide both sides by 4.*

$y = -1$

Step 4 $(2, -1)$ — *Write the solution as an ordered pair.*

Check It Out! Example 1

Solve $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$ by elimination.

Step 1

$$\begin{array}{rcl} y + 3x &=& -2 \\ 2y - 3x &=& 14 \\ \hline \end{array}$$

Align like terms. 3x and −3x are opposites.

Step 2

$$3y + 0 = 12$$

Add the equations to eliminate x.

$$3y = 12$$

Simplify and solve for y.

$$\frac{3y}{3} = \frac{12}{3}$$

Divide both sides by 3.

$$y = 4$$

Check It Out! Example 1 Continued

Step 3 $y + 3x = -2$ *Write one of the original equations.*

$4 + 3x = -2$ *Substitute 4 for y.*

$\underline{-4} \qquad \underline{-4}$ *Subtract 4 from both sides.*

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$ *Divide both sides by 3.*

$x = -2$

Step 4 $(-2, 4)$ *Write the solution as an ordered pair.*

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of *each* term.

Example 2: Elimination Using Subtraction

Solve $\begin{cases} 2x + y = -5 \\ 2x - 5y = 13 \end{cases}$ **by elimination.**

Step 1

$$\begin{aligned} 2x + y &= -5 \\ -(2x - 5y &= 13) \\ \hline 2x + y &= -5 \\ -2x + 5y &= -13 \\ \hline \end{aligned}$$

Both equations contain 2x. Add the opposite of each term in the second equation.

Step 2

$$\begin{aligned} 0 + 6y &= -18 \\ 6y &= -18 \\ y &= -3 \end{aligned}$$

Eliminate x.

Simplify and solve for y.

Example 2 Continued

Step 3 $2x + y = -5$ — *Write one of the original equations.*

$2x + (-3) = -5$ — *Substitute –3 for y.*

$2x - 3 = -5$

$\underline{+3} \quad \underline{+3}$ — *Add 3 to both sides.*

$2x = -2$ — *Simplify and solve for x.*

$x = -1$

Step 4 $(-1, -3)$ — *Write the solution as an ordered pair.*

Remember!

Remember to check by substituting your answer into both original equations.

Check It Out! Example 2

Solve $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$ by elimination.

Step 1

$$\begin{array}{r} 3x + 3y = 15 \\ -(-2x + 3y = -5) \\ \hline \end{array}$$

$$\begin{array}{r} 3x + 3y = 15 \\ +\ 2x - 3y = +5 \\ \hline \end{array}$$

Both equations contain 3y. Add the opposite of each term in the second equation.

Step 2

$$5x + 0 = 20$$

Eliminate y.

$$5x = 20$$

Simplify and solve for x.

$$x = 4$$

Check It Out! Example 2 Continued

Step 3 $3x + 3y = 15$ — *Write one of the original equations.*

$3(4) + 3y = 15$ — *Substitute 4 for x.*

$$\begin{aligned} 12 + 3y &= 15 \\ \underline{-12} \quad\;\; &\; \underline{-12} \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

Subtract 12 from both sides.

Simplify and solve for y.

Step 4 $(4, 1)$ — *Write the solution as an ordered pair.*

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients.

Example 3A: Elimination Using Multiplication First

Solve the system by elimination.

$$\begin{cases} x + 2y = 11 \\ -3x + y = -5 \end{cases}$$

Step 1

$$\begin{array}{r} x + 2y = 11 \\ -2(-3x + y = -5) \\ \hline x + 2y = \quad 11 \\ +(6x - 2y = +10) \\ \hline 7x + \quad 0 = 21 \end{array}$$

Multiply each term in the second equation by –2 to get opposite y-coefficients.

Add the new equation to the first equation to eliminate y.

Step 2

$$7x = 21$$

$$x = 3$$

Solve for x.

Example 3A Continued

Step 3 $x + 2y = 11$ — *Write one of the original equations.*

$3 + 2y = 11$ — *Substitute 3 for x.*

$\underline{-3} \qquad \underline{-3}$ — *Subtract 3 from both sides.*

$2y = 8$ — *Solve for y.*

$y = 4$

Step 4 $(3, 4)$ — *Write the solution as an ordered pair.*

Example 3B: Elimination Using Multiplication First

Solve the system by elimination.

$$\begin{cases} -5x + 2y = 32 \\ 2x + 3y = 10 \end{cases}$$

Step 1

$$2(-5x + 2y = 32)$$
$$5(2x + 3y = 10)$$

Multiply the first equation by 2 and the second equation by 5 to get opposite x-coefficients

$$-10x + 4y = 64$$
$$+(10x + 15y = 50)$$

Add the new equations to eliminate x.

Step 2

$$19y = 114$$

$$y = 6$$

Solve for y.

Example 3B Continued

Step 3 $2x + 3y = 10$ — *Write one of the original equations.*

$2x + 3(6) = 10$ — *Substitute 6 for y.*

$2x + 18 = 10$

$\underline{-18} \quad \underline{-18}$ — *Subtract 18 from both sides.*

$2x = -8$

$x = -4$ — *Solve for x.*

Step 4 $(-4, 6)$ — *Write the solution as an ordered pair.*

Check It Out! Example 3a

Solve the system by elimination.

$$\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$$

Step 1

$$\begin{array}{r} 3x + 2y = 6 \\ 3(-x + y = -2) \\ \hline 3x + 2y = 6 \\ +(-3x + 3y = -6) \\ \hline 0 \quad + 5y = 0 \end{array}$$

Multiply each term in the second equation by 3 to get opposite x-coefficients.

Add the new equation to the first equation.

Step 2

$$5y = 0$$

$$y = 0$$

Simplify and solve for y.

Check It Out! Example 3a Continued

Step 3	$-x + y = -2$	*Write one of the original equations.*
	$-x + 3(0) = -2$	*Substitute 0 for y.*
	$-x + 0 = -2$	*Solve for x.*
	$-x = -2$	
	$x = 2$	
Step 4	$(2, 0)$	*Write the solution as an ordered pair.*

Check It Out! Example 3b

Solve the system by elimination.

$$\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$$

Step 1

$$3(2x + 5y = 26)$$
$$+(2)(-3x - 4y = -25)$$

$$6x + 15y = 78$$
$$+(-6x - 8y = -50)$$

Multiply the first equation by 3 and the second equation by 2 to get opposite x-coefficients

Step 2

$$0 + 7y = 28$$

Add the new equations to eliminate x.

$$y = 4$$

Solve for y.

Check It Out! Example 3b Continued

Step 3 $2x + 5y = 26$ — *Write one of the original equations.*

$2x + 5(4) = 26$ — *Substitute 4 for y.*

$$\begin{aligned} 2x + 20 &= 26 \\ -20 \quad &\quad -20 \\ \hline 2X \quad &= 6 \end{aligned}$$

Subtract 20 from both sides.

$x = 3$ — *Solve for x.*

Step 4 $(3, 4)$ — *Write the solution as an ordered pair.*

Example 4: *Application*

Paige has $7.75 to buy 12 sheets of felt and card stock for her scrapbook. The felt costs $0.50 per sheet, and the card stock costs $0.75 per sheet. How many sheets of each can Paige buy?

Write a system. Use *f* for the number of felt sheets and *c* for the number of card stock sheets.

$0.50f + 0.75c = 7.75$ *The cost of felt and card stock totals $7.75.*

$f + c = 12$ *The total number of sheets is 12.*

Example 4 Continued

Step 1

$$0.50f + 0.75c = 7.75$$
$$+ (-0.50)(f + c) = 12$$

Multiply the second equation by –0.50 to get opposite f-coefficients.

$$0.50f + 0.75c = 7.75$$
$$+ (-0.50f - 0.50c = -6)$$

Add this equation to the first equation to eliminate f.

Step 2

$$0.25c = 1.75$$

$$c = 7$$

Solve for c.

Step 3

$$f + c = 12$$

Write one of the original equations.

$$f + 7 = 12$$

Substitute 7 for c.

$$\underline{-7 \quad -7}$$

Subtract 7 from both sides.

$$f = 5$$

Example 4 Continued

Step 4 (7, 5) *Write the solution as an ordered pair.*

Paige can buy 7 sheets of card stock and 5 sheets of felt.

Check It Out! Example 4

What if…? Sally spent \$14.85 to buy 13 flowers. She bought lilies, which cost \$1.25 each, and tulips, which cost \$0.90 each. How many of each flower did Sally buy?

Write a system. Use l for the number of lilies and t for the number of tulips.

$1.25l + 0.90t = 14.85$ *The cost of lilies and tulips totals \$14.85.*

$l + t = 13$ *The total number of flowers is 13.*

Check It Out! Example 4 Continued

Step 1

$$
\begin{array}{r}
1.25l + .90t = 14.85 \\
+ \; (-.90)(l + t) = 13 \\
\hline
\end{array}
$$

Multiply the second equation by –0.90 to get opposite t-coefficients.

$$
\begin{array}{r}
1.25l + 0.90t = 14.85 \\
+ \; (-0.90l - 0.90t = -11.70) \\
\hline
0.35l = 3.15
\end{array}
$$

Add this equation to the first equation to eliminate t.

Step 2

Solve for l.

$$l = 9$$

Check It Out! Example 4 Continued

Step 3

$$l + t = 13$$

$$\begin{aligned} 9 + t &= 13 \\ \underline{-9} \quad & \quad \underline{-9} \\ t &= 4 \end{aligned}$$

Write one of the original equations.

Substitute 9 for l.

Subtract 9 from both sides.

Step 4 (9, 4)

Write the solution as an ordered pair.

Sally bought 9 lilies and 4 tulips.

All systems can be solved in more than one way. For some systems, some methods may be better than others.

Systems of Linear Equations

METHOD	USE WHEN...	EXAMPLE
Graphing	• Both equations are solved for y. • You want to estimate a solution.	$\begin{cases} y = 3x + 2 \\ y = -2x + 6 \end{cases}$
Substitution	• A variable in either equation has a coefficient of 1 or −1. • Both equations are solved for the same variable. • Either equation is solved for a variable.	$\begin{cases} x + 2y = 7 \\ x = 10 - 5y \end{cases}$ or $\begin{cases} x = 2y + 10 \\ x = 3y + 5 \end{cases}$
Elimination	• Both equations have the same variable with the same or opposite coefficients. • A variable term in one equation is a multiple of the corresponding variable term in the other equation.	$\begin{cases} 3x + 2y = 8 \\ 5x + 2y = 12 \end{cases}$ or $\begin{cases} 6x + 5y = 10 \\ 3x + 2y = 15 \end{cases}$

Lesson Quiz

Solve each system by elimination.

1. $\begin{cases} 2x + y = 25 \\ 3y = 2x - 13 \end{cases}$ (11, 3)

2. $\begin{cases} -3x + 4y = -18 \\ x = -2y - 4 \end{cases}$ (2, –3)

3. $\begin{cases} -2x + 3y = -15 \\ 3x + 2y = -23 \end{cases}$ (–3, –7)

4. Harlan has \$44 to buy 7 pairs of socks. Athletic socks cost \$5 per pair. Dress socks cost \$8 per pair. How many pairs of each can Harlan buy?

4 pairs of athletic socks and 3 pairs of dress socks